**Q.** How does the arithmetic and calculus of power series (as functions) work?

**Thm:** (Combining/Modifying Power Series) Suppose $f(x) = \sum_{n=0}^{\infty} c_n x^n$ converges for all $x$ in _____ and $g(x) = \sum_{n=0}^{\infty} d_n x^n$ converges for all $x$ in _____. Let $R =$_____. Then

(a) The series $\sum_{n=0}^{\infty} (c_n \pm d_n)x^n$ converges to $f(x) \pm g(x)$ for all _____.

(b) Given a constant $b$ and an integer $m \geqslant 0$, the series $\sum_{n=0}^{\infty} bc_n x^{n+m} = \sum_{n=0}^{\infty} (bx^m)c_n x^n$ converges to $bx^m f(x)$ for all $x$ in $(-R, R)$.

(c) Given a constant $b$ and an integer $m \geqslant 0$, the series $\sum_{n=0}^{\infty} b^n c_n x^{mn} = \sum_{n=0}^{\infty} c_n (bx^m)^n$ converges to $f(bx^m)$ for all $x$ such that $bx^m \in (-R, R)$.

(d) If $e_n = \sum_{k=0}^{n} c_k d_{n-k} = c_0 d_n + c_1 d_{n-1} + \cdots + c_n d_0$, then the series $\sum_{n=0}^{\infty} e_n x^n$ converges to the product $f(x) \cdot g(x)$ for all $x \in (-R, R)$.

**Q.** What does this theorem mean?

**Ex 1** Suppose $f(x) = \dfrac{1}{5-x}$ and $g(x) = \dfrac{1}{x-2}$.

(a) Find the power series for $f$ and $g$ centered at 0.

(b) Use part (a) to find the power series for $\dfrac{1}{5-3x^7}$ centered at 0.

(c) Use part (a) to find the power series for $\dfrac{3x}{x^2-2}$ centered at 0.

(d) Write out the first 3 terms of the series for $\dfrac{1}{(5-x)(x^2-2)}$ centered at 0.

**Ex 2** Find the series $\dfrac{1}{(5-x)(x-2)}$ centered at 0 using partial fraction decomposition.

**Q.** Suppose that $\sum_{n=0}^{\infty} c_n x^n$ has radius of convergence $R$. What is the radius of convergence of $\sum_{n=1}^{\infty} n \cdot c_n x^{n-1}$? What about $\sum_{n=0}^{\infty} \frac{c_n}{n+1} x^{n+1}$? Can you tell anything about the intervals of convergence for these two series?

**Thm:** (Term-by-term Differentiation and Integration) Suppose $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$ converges for all _____. Then for all _____, $f$ is

_____ and

**Ex 3** Find the series for $f(x) = \dfrac{1}{(1-x)^2}$ centered at 0.

**Ex 4** Find the series for $f(x) = \ln|1-x|$ centered at 0.

**Ex 5** Use power series to estimate ln(2).